The demand
demand for risky asset-demand for insurance

The previous chapters gave a precise definition of risk-aversion. The aim of this chapter is to show that this particular behaviuour can explain the main characteristics of demand for "insurance" and for "financial assets". The two models that we describe here are formally equivallent. The first one deals with insurance demand. Insurance occurs when one party agrees to pây an indemnity to another party in case of occurence of a random event generating a loss. The most common example is insurance policy : an insurance company, the insurer, commits to pay an indemnity and is compensated by being payed a fix premium by the policyholder. Several other contracts, in the economic life, involve a part of insurance. For instance, a contract that specifies a fixed wage for an employee, whatever the level of the profits of the firm, insures one party of the firm's stakeholders.

It is important to remark that the availability of insurance contracts has a huge importance for economic growth. Without insurance people would be reluctant to take risky decisions. In our example of Bill and George, Bill alone could have been refreined to take risk and to invest in his project. Sharing the risk with George strengthen the incentive to take the risk.

The second models deals with the demand for risky assets. Financial markets are the places where "entrepreneurial" risks are exchanged. Participants in these markets are risk-averse people who accepts to take risks only if they receive appropriate awards for it. Here, taking a risk is compensated by a higher expected return. What we call "equity premium" represents the premium that investors are willing to get to accept risk.

These two behaviour are hence linked to risk aversion and the aim of this chaper is to describe the main features of demand.

1 The demand for insurance

Take an individual with an initial wealth w_0 bearing a loss $\widetilde{x}$. The lottery on his wealth he faces is hence : $\widetilde{y}(0) = w_0 - \widetilde{x}$. Here $\widetilde{x}$ is a positive random variable. Suppose that this consumer can buy an insurance contracts that specifies both a premium and an indemnity schedule.

Definition 1 *An insurance contract is defined by*

- a fixed premium π

- an indemnity schedule $q(x)$ that gives the indemnity payed by the insurer in case of loss dx.

When he buys the contract $z = (\pi, q)$, The policy holder's wealth becomes $\widetilde{y}(z) = w_0 - \widetilde{x} + q(\widetilde{x}) - P$.

In the real life the function q can take several different forms among which two are remarkable.

Definition 2 *When q is linear $q = \beta x$, insurer agrees to reimburse only a fraction β of the loss. $(1-\beta)$ is called the co-insurance rate. When $\beta = 1$ we say that there is a full coverage.*

When q is such that the F first dollars are not insured, $q = \max(0, x-F)$, F is called "straight deductible".

1.1 The demand in the linear case

We suppose in this section that indemnity is linear $q = \beta x$. For the insurer the expected cost of such a contract is $\beta E(\widetilde{x})$. $\beta E(\widetilde{x})$ is called the "fair" or "actuarial premium" of the contract. A risk neutral insurer who propose a premium $\beta E(\widetilde{x})$ woud make zero expected profit. Suppose that "transaction costs" are such that the total expected cost of the contract is $(1+\lambda)\beta E(\widetilde{x})$, and that the insurer proposes a loaded premium equal to this cost. Then the lottery faced by the policyholder if he chooses the coinsurance level β will be :

$$\widetilde{y}(\beta) = w_0 - \widetilde{x} + \beta\widetilde{x} - (1+\lambda)\beta E(\widetilde{x})$$

What is therefore the optimal insurance rate? Let us solve the following problem :

$$\max_{\beta} E\left[u(w_0 - \widetilde{x} + \beta\widetilde{x} - (1+\lambda)\beta E(\widetilde{x}))\right]$$

Take the function $H(\beta) = E\left[u(\widetilde{y}(\beta))\right]$. We have $H'(\beta) = E\left[u'\left(\widetilde{y}(\beta)\right)\widetilde{y}'(\beta)\right]$ and, because $\widetilde{y}''(\beta)$ is nul, $H''(\beta) = E\left[u''\left(\widetilde{y}(\beta)\right)\left(\widetilde{y}'(\beta)\right)^2\right]$. Since u is concave u'' is negative and then H'' is negative. H is hence a concave function : if it has a maximum, it is such that $H'(\beta^*) = 0$.

Take $\beta = 1$ we have :

$$H'(1) = -u'(w_0 - (1+\lambda)E(\widetilde{x}))\lambda E(\widetilde{x}) \leq 0$$

Hence, as soon as $\lambda > 0$, partial coverage $(\beta < 1)$ is optimal since $H'(1) < 0$ and the concavity of H imply that the optimum β^* is smaller than 1. If conversely $\lambda = 0$ then the optimal coverage is $\beta^* = 1$.

Proposition 3 *When the loading factor is nul, the optimal coverage rate is $\beta = 1$: a fair premium leads insuree to demand complete insurance. When the loading factor is positive then the deman involves partial insurance $\beta < 1$.*

Another question concerns the variation of β^* when risk aversion or initial risk-free income vary.

The intuition suggests that when risk aversion increases then the demand for coverage increases. Take u_1 and u_2 two utility functions such that u_2 is more risk-averse than u_1. There exists an increasing concave function such that $u_2 = \phi \circ u_1$. Take β_1^* the optimal level of insurance for the first agent : it is such that $H_1'(\beta_1^*) = 0$.. We have $H_2'(\beta_1^*) = E(u_2'(\widetilde{y}(\beta_1^*))(\widetilde{x} - (1+\lambda)E(\widetilde{x})) = E\left[\left(\phi'(u_1(\widetilde{y}(\beta_1^*))\right)(u_1'(\widetilde{y}(\beta_1^*))(\widetilde{x} - (1+\lambda)E(\widetilde{x})))\right]$

Note : $\phi'(u_1(\widetilde{y}(\beta_1^*)) = A(\widetilde{x})$ and $u_1'(\widetilde{y}(\beta_1^*))(\widetilde{x} - (1+\lambda)E(\widetilde{x})) = B(\widetilde{x})$ It is easy to see that A is a positive increasing function.

We have $H_2'(\beta_1^*) = E\left(A(\widetilde{x})B(\widetilde{x})\right)$ and the first order condition for the optimality of β_1^* is $E(B(\widetilde{x})) = 0$.

$\frac{E(A(\widetilde{x})B(\widetilde{x}))}{E(A(\widetilde{x}))}$ is a new mean of $B(\widetilde{x})$ in which the weight of large values is greater tha the one with the initial density. Hence, As $E(B(\widetilde{x})) = 0$, then $E\left(A(\widetilde{x})B(\widetilde{x})\right) = H_2'(\beta_1^*) \geq 0$ and $\beta_2^* \geq \beta_1^*$.

Proposition 4 *When risk aversion increases, the demand for coverage increases (untill it is less than one).*

What does happen when the initial wealth increases? To make expressions more readable set $(1+\lambda)E(\widetilde{x}) = P$. Set $\beta(w_0)$ the level of coverage chosen by an individual with an initial wealth w_0. Set $H(w_0, \beta) = E\left[u(w_0 - \widetilde{x} + \beta\widetilde{x} - \beta P)\right]$ and note H' the derivative of this function with respect to β.

Remark that $\frac{\partial H'}{\partial w_0} = E\left[u''(\widetilde{y}(\beta))(\widetilde{x} - P)\right] = -\frac{\partial E(-u'(\widetilde{y}(\beta)))}{\partial \beta}$. Take the agent with the utility function $v = -u'$. We have seen that when u exhibits DARA, (Decreasing Absolute Risk Aversion) then $-u'$ is more concave than u. That means that the optimal coverage for such a more risk-averse individual is larger which means that $\frac{\partial E(-u'(\widetilde{y}(\beta)))}{\partial \beta}$ evaluated at $\beta(w_0)$ is positive. This in turn implies $\frac{\partial H'}{\partial w_0} < 0$. Moreover, $\beta(w_0)$ is such that $H'(w_0, \beta(w_0)) = 0$ which implies $H'' \frac{\partial \beta}{\partial w_0} + \frac{\partial H'}{\partial w_0} = 0$. And hence $\frac{\partial \beta}{\partial w_0} \leq 0$.

Proposition 5 *When the decision maker has decreasing absolute risk aversion, then the optimal coverage is decreasing when w_0 increases.*

1.2 The optimality of deductibles

The linear case, examined in the previous section is somewhat particular. A natural question arises to look for the optimal form of the indemnity function. Take the contract $(\pi,\ q(.))$ where q is the function that specifies the indemnity when the loss is x. We must have $q(x) \geq 0$ and $q(x) \leq x$. This last conditions is needed if we want to avoid moral hazard problems : if it were not the case, there would be a strong incentive for the insuree to provoque the loss. If the insurer incurs administrative costs the premium charged must be such that $\pi \geq (1+\lambda)E(q(x))$. The problem hence writes :

$$\max_{q(.)} E(u(w_0 - \widetilde{x} + q(\widetilde{x}) - \pi), \pi \geq (1+\lambda)E(q(x)), 0 \leq q(x) \leq x$$

If we suppose that x is distributed according to a density function dF we can write the problem as a variational one :

$$\left| \begin{array}{c} \max\limits_{q(.),\pi} \int u(w_0 - s + q(s) - \pi) dF(s) \\ \int (\pi - (1+\lambda)q(s)) dF(s) \geq 0 \\ \forall x, q(x) \geq 0 \\ \forall x, x \geq q(x) \end{array} \right.$$

The Lagrangian of this problem writes :

$$\begin{aligned} \mathcal{L} = \int \{ & u(w_0 - s + q(s) - \pi) \\ & + \mu(\pi - (1+\lambda)q(s)) \\ & + \nu(s)q(s) \\ & + \tau(s)(s - q(s)) \} dF(s) \end{aligned}$$

Where $\mu \geq 0$ is the Lagrange multiplier assocuiated to the insurer break-even constraint, and $\nu(s) \geq 0$ and $\tau(s) \geq 0$ the Euler Lagrange multipliers associated to the other constraints.

The first order conditions are :

$$u'(w_0 - x + q(x) - \pi) - \mu(1+\lambda) + \nu(x) - \tau(x) = 0 \quad (1)$$
$$\int \{-u'(w_0 - s + q(s) - \pi) + \mu\} dF(s) = 0 \quad (2)$$
$$\mu \int (\pi - (1+\lambda)q(s)) dF(s) = 0 \quad (3)$$
$$\nu(x)q(x) = 0 \quad (4)$$
$$\tau(x)(x - q(x)) = 0 \quad (5)$$

To solve thes equations, first look for values of the loss such that $0 < q(x) < x$. For those values E4 and E5 imply $\nu(x) = \tau(x) = 0$. E1 gives :

$$u'(w_0 - x + q(x) - \pi) = \mu(1 + \lambda)$$

which means that (since the right hand side is independant of x) $w_0 - x + q(x) - \pi$ is independant of x. Hence there exists D such that $q(x) = x - D$. The value of D must be such that by $u'(w_0 - D - \pi) = \mu(1 + \lambda)$. Define I the set of x. such that $0 < q(x) < x$, obviously $I \subset]D, +\infty[$.

What are the values for which $q(x) = x$ (and $x \neq 0$)? For those values we must have :

$$u'(w_0 - \pi) - \mu(1 + \lambda) - \tau(x) = 0$$

That implies that $\tau(x)$ is constant $\tau(x) = u'(w_0 - \pi) - \mu(1 + \lambda)$. which is not compatible with the existence of a positive value of D previously defined. except if $D = 0$.

It comes that the solution is of the form : $q(x) = x - D$ for $x \in I$ and $q(x) = 0$ for $x \notin I$

It is now easy to show that $q(x) = (x - D)^+$ is solution. (let to the reader).