The Geneva Papers on Risk and Insurance Theory, 27: 87–113, 2002

Reinsuring Climatic Risk Using Optimally Designed Weather Bonds

PAULINE BARRIEU p.m.barrieu@lse.ac.uk
Université de Paris VI, Laboratoire de Probabilités, 173 rue du Chevaleret, 75013 Paris, France;
Doctorat HEC, Groupe HEC, 1 rue de la Libération, 78351 Jouy-en-Josas Cédex, France

NICOLE EL KAROUI elkaroui@cmapx.polytechnique.fr
C.M.A.P., Ecole Polytechnique, 91128 Palaiseau Cédex, France

Received June 26, 2001; Revised October 4, 2002

Abstract

The aim of this paper is to determine the optimal structure of a weather bond, i.e. a bond whose coupons depend on the occurence of a weather event. The stress is put more on the structuration than on the simple pricing of the bond. Therefore, instead of looking only at the bond issue, we consider it as a part of a more general transaction, involving three agents: a firm, which wants to be hedged against its weather risk, an investor, which buys the bond and a bank, which has an intermediary key role. Then, we derive the optimal characteristics of the whole transaction. But the bond structure which is obtained, corresponds to a minimal structure: indeed, only the bond optimal price function and its optimal reimbursement level (amount which is paid back when an event occurs) can be determined while there is a degree of freedom in the choice of the optimal coupon. Therefore, this indeterminacy may be interpreted as a marketing tool and it could play an important role in the negotiation process between the issuer and the investor.

Key words: non-financial risk, weather bond, utility maximization, optimal design, insurance

JEL Classification No.: C61, D81, G13, G22

1. Introduction

Since 1997 the United States has witnessed the arrival of a new breed of financial assets: weather derivatives. They allow firms to manage the climatic risk which disturbs their activities and may notably entail a variability of earnings and costs.

These new but illiquid instruments have a structure which is relatively standard: they are options, swaps or bonds. They depend, however, on the evolution of very particular underlying assets since they are related to the weather (temperature, precipitation, wind...). These assets are not quoted on markets and are not replicable. Therefore, a standard risk-neutral point of view is not well-suited and the weather derivatives' market can be considered as "acomplete" in the sense that only derivatives are traded on it.

Another particularity of these products is the problem of their classification (see Geman [1999]): weather derivatives are financial products by their structure but insurance products by their logic. This difficulty of classification is a common factor to the whole A.R.T. ("Alternative Risk Transfer") business i.e. the securitization of insurance traditional risks:

as Farny [1999] writes there is no clear and obvious distinction between insurance and finance when these new types of transaction are concerned. This ambiguity is especially clear for the weather bond: this is a standard bond apart from the fact that its coupons depend on the occurrence of a weather event. In that sense, this is a financial product. But, its origins—the firm's demand for a hedge against a climatic risk—or the diversification of the issuer's risk on small bondholders can make us think more of an insurance policy or of an agreement of mutual assistance (see Gallix [1985]).

This paper focuses on an accurate analysis of a weather bond i.e. on the joint determination of its coupon level and its price. This particular study helps us to better define the characteristics and stakes of this new market. Moreover, the problems relative to the weather bond's characterization are not obvious, as underlined by the two and only emission's attempts of autumn 1999, which have failed: Enron's emission was cancelled and that of Koch was reduced by half, for lack of buyers. These difficulties are all the more surprising so since cat bonds, whose logic is quite similar (i.e. securitization of a non-financial risk), are relatively successful.

Furthermore the aim of this article is neither to determine a dynamic for weather data nor to propose a prediction model. Indeed, many articles have been interested in these questions: especially, Dischel [1998, 1999, 2000a, 2000b], Cao and Wei [2000] or Dornier and Queruel [2000] try to propose a statistical model for temperature. Moreover, a joint analysis, coupon-price, of a (standard) bond appears to be original: indeed, bonds studied in the literature often are zero-coupon bonds (as Longstaff and Schwartz [1995] or Briys [1998a, 1998b]) or their coupon allows their emission to be at par. Very recently, Sankaran [2000] has proposed a model for weather bond pricing. However, in this article, the coupon levels are taken a priori and the author focuses on the characterization of a model for the underlying temperature data. The bond price is given by the classical method of net present value. Even so the role of weather bond coupons is not simple: they take part in the structure of the global transaction and have, for that reason, their own purpose. Therefore, the characterization of the optimal bond structure, and not only the simple determination of its price, is the core matter of this study.

Hence, after having specified the notations and the assumptions used in this paper, we propose a modelling for the global transaction, involving the definition of a choice criterion for the different agents: the firm, the bank and the investor. Solving the optimization problem related to the global transaction leads us to the study of both the insurance relation between the firm and the bank and the bond issue. Looking at the origins of the bond emission helps us to better understand the specificities of the weather bond coupons. Finally, the optimal structure of the emission i.e. the bond price function and the optimal amount, which is paid back if an event occurs, are eventually determined.

2. General presentation, assumptions and notations

2.1. *Description of the transaction*

This paper focuses on the analysis of a weather bond. The principal is assumed to be non-risky whereas the payment of the coupons depends on the occurrence of a given weather event. More precisely, the amount of the coupon is reduced when an event occurs. In this case, the bondholders receive less than in the situation when nothing happens.

However, considering such a bond without taking into account the origins of its emission is a myopic attitude. Indeed, the characteristics of this bond (coupons, amount which is paid back when an event occurs, price) cannot be chosen arbitrarily. They have to fully play their risk diversification role for the issuer: everything starts when a firm is facing a climatic risk and calls on a bank to be hedged against the fateful effects of weather on its activities. For a premium, the latter commits itself to transferring compensation to the firm if loss obtains. Then, the bank issues a weather bond, so that it can diversify its new risk on small bondholders. For that reason, the whole story will be taken into account and the bond emission will be viewed as a part of a more general transaction.

As a summary, this transaction has the following structure:

– If no event occurs during the bond's life, the flows' structure is given by figure 1, where *S.P.V.* denotes Special Purpose Vehicle. This is a legal entity, independent of bank's other activities.
– If, on the contrary, a weather event occurs (one possibility per year, non-exclusive), two additional flows appear (see figure 2). Thus, this transaction involves three agents: a firm, a bank and an investor. Such a structure is quite classical when the securitisation of a non-financial risk is at stake. A direct relationship between the firm and the investor is usually unfeasible since the firm does not often have the same qualifications as the bank especially for risk tranfer.

This paper aims to characterize the structure of the weather bond. We particularly focus on its coupons. In fact, contrary to more standard bonds, the coupon plays a full role here. It has its own purpose: indeed, its function is to design not a "nice-looking" emission (for example, emission at par) but the global transaction structure. It plays a key role in the compensation program of the bank. Fixing its level arbitrarily may be very hazardous as

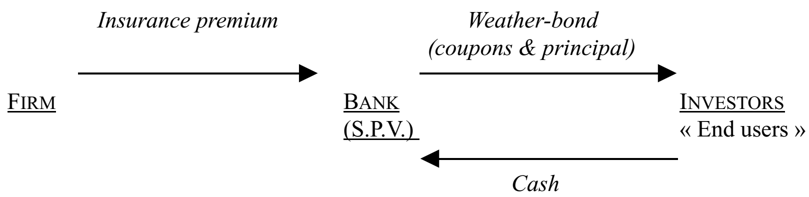

Figure 1. Flows of the transaction.

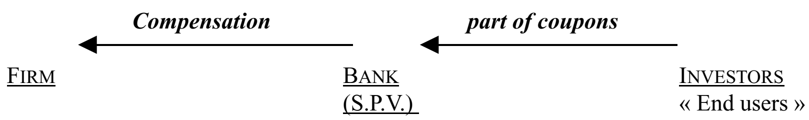

Figure 2. Additional flows when a weather event occurs.

it may be inadequate with respect to that of the compensation. The coupon level has to be determined so that the global transaction is not unfavourable to the bank.

Moreover, the presence of three actors (the firm, the bank and the investor) plays a key role in the characterization of the different parameters of the global transaction and especially of the bond emission, as we will see in the next sections.

2.2. Assumptions and notations

The time period considered in this article is n years corresponding to the bond maturity. Each year is indexed by i, $i \in \{0; 1; \dots; n\}$ and $\beta_{i,n}$ represents the capitalization factor of year i to year n. Especially, we have

$$\beta_{n,n} = 1$$

In the following, we refer only to β_i as $\beta_{i,n}$ since all the flows are capitalized till year n, the bond maturity. Flows are not reinvested.

The risk inherent to the transaction, i.e. the occurrence of a weather event, is considered in this article as the only risk in the market. It is modelled as a family of random variables. We refer to ε_i as the random event of year i ($i \neq 0$). An event occurs if $\mathbf{1}_{\varepsilon_i} = 1$. In the following, ε_i designates indifferently both the random variable and its occurrence variable. This latter is a Bernoulli variable, as it can take only values in $\{0; 1\}$. We do not make any assumptions for the independence of the ε_i and for their respective parameter p_i under the statistical probability $\mathbb{P}$. In the following, $\mathbb{E}$ denotes the expected value with respect to $\mathbb{P}$. The aim of this paper is not an accurate study of weather data and their distribution. For that reason, p_i and the correlation between the different variables are assumed to be known and we do not focus on their calculation.

Three agents are considered in this study. They are linked together by the financial structure of future cash flows. Indeed, each of them has the following financial commitments: *A firm, denoted as agent F*, is facing a weather risk. Its risk can be broken down into different losses shared out among several years. It is characterized by the random variable Θ, which may be defined, for instance, by

$$\Theta = M \sum_{i=1}^{n} \mathbf{1}_{\varepsilon_i} \beta_i = M \sum_{i=1}^{n} \varepsilon_i \beta_i \tag{1}$$

where M is year i loss amount ($i \neq 0$), if an event ε_i occurs during this particular year. M is assumed to be constant. In this particular framework, Θ can only take a finite number of values (indeed, it can take 2^n possible different values). As it is not a determining factor in the results of this article, the general notation Θ will be kept as far as possible. The firm calls on *a bank, denoted as agent B*, to be hedged against this risk and pays an amount, π, in year 0, to be protected against the risk of loss. In exchange, it receives a compensation $J(\theta)$ if loss $\Theta = \theta$ obtains. As an insurance reimbursement is necessarily nonnegative and cannot exceed the size of the loss, the coverage function must satisfy the following constraint:

$$0 \leq J(\theta) \leq \theta \quad \forall \theta \in \mathcal{D}(\Theta) \tag{2}$$

where $\mathcal{D}(\Theta)$ is the support of the law of Θ i.e. in the discrete case, the set of all possible values taken by Θ, with a positive probability. In the following, the definition set of $J(\Theta)$ is denoted as $\mathcal{D}(\Theta)$.
Moreover, this constraint can also be written in terms of the random variable Θ as

$$0 \leq J(\Theta) \leq \Theta \quad \mathbb{P}\, a.s.$$

Remark 1 (On the compensation function)**:** In this paper, the risk of loss is assumed to be perfectly covered by the compensation. Indeed, the same source of risk Θ intervenes in both the exposure of the firm and the compensation paid by the bank. Such an assumption[1] does not take into account any basis risk, which may remain even after an insurance coverage. This particular aspect will be precised later.

Some authors have considered the impact of basis risk and moral hazard on the optimal insurance strategy (see, for instance, Doherty and Mahul [2001]).

Hence, the flows, related to the "insurance" part of the transaction and capitalized from the moment when they occur to year n, can be written for both agents F and B as follows

$$\text{For agent } F, \quad -\pi\beta_0 - \Theta + J(\Theta) \tag{3}$$

$$\text{For agent } B, \quad \pi\beta_0 - J(\Theta) \tag{4}$$

By assumption, in this part of the transaction, agent B focuses only on its relation with the firm. It does not know how to manage its risk yet.

To diversify its risk, agent B decides to issue a weather bond. *The investor*, buying this bond, is denoted as *agent I*. It pays the bond price Φ to agent B in year 0. In exchange, it receives, each year i, a coupon s and, in year n, the principal N. Since the coupons are subject to weather risk, agent I has to pay back a constant amount α to agent B, when an event occurs. This amount is considered as an entity itself and do not relate it directly to the coupon s. This particular point will be discussed later.

Hence, the flows, related to the bond emission and capitalized from the moment when they take place to year n, can be written for both agents B and I as follows

$$\text{For agent } B, \quad \pi\beta_0 - J(\Theta) + \Phi\beta_0 - s\sum_{i=1}^{n}\beta_i - N + \frac{\alpha}{M}\Theta \tag{5}$$

$$\text{For agent } I, \quad -\Phi\beta_0 + s\sum_{i=1}^{n}\beta_i + N - \frac{\alpha}{M}\Theta \tag{6}$$

For the sake of simplicity, Λ will denote in the following the non-random flows (or the amount of cash[2]) received by the investor and capitalized to year n

$$\text{For agent } B, \quad \pi\beta_0 - J(\Theta) - \Lambda + \frac{\alpha}{M}\Theta$$

$$\text{For agent } I, \quad \Lambda - \frac{\alpha}{M}\Theta$$

This notation is completely equivalent to the first one as only the expression $\Lambda = -\Phi\beta_0 + s\sum_{i=1}^{n}\beta_i + N$ intervenes in the representation of this part of the transaction. No particular role is played by s, N or Φ, taken separately. This point will be developed later.

In this second part of the transaction, agent B has a more global view: it takes into account both its relations with agent F and agent I.

3. Modelling the choice criterion for the characteristics of the global transaction

As underlined previously, the bond emission can be considered as an element of a more global transaction, analyzing and studying this transaction give us a key to better understand the bond emission and to value it. Moreover, this transaction involves three different agents, which play different roles at different stages of the deal. However, as previously underlined, it can be divided into two smaller transactions: agent F is at the origins of the first one (and, for that reason, of the global one). It calls on agent B for a protection against a risk of loss. The second transaction is consecutive to the first one: indeed, agent B transfers the risk, it is now bearing, on agent I, by issuing a weather bond. Hence, the global problem of the characterization of the transaction is naturally set in two subproblems, each of them representing a smaller transaction.

Moreover, in this section, the multiplicity of roles playing by agent B is particularly taken into account. It has not only a classical function of a banker towards agent I but also an insurer's role towards agent F. This schizophrenia emphasizes one of the weather derivatives' features, as they lie midway between finance and insurance.

3.1. Risk aversion and utility criterion

All agents (B, F and I) are assumed to be risk-averse, even the bank. Such an assumption may be justified by the risk specificity. Since Θ is the only source of risk, which is taken into account in this study, it cannot be reduced by diversification. The attitude of the different agents towards risk is modelled via a utility function. Such a function is supposed to represent the level of satisfaction a given economic agent gets from a given situation. It depends on its risk-aversion. All agents are usually assumed to be rational, moreover they want to maximize the utility they can expect from a future (and uncertain) situation. A choice criterion for a given agent may be the maximization of its expected utility. Hence, in this rational framework, some conditions on utility functions are required: they have to be continuous, strictly increasing and concave.

For the sake of simplicity in this particular study, as no particular constraint is imposed on the different capitalized flows, utility functions of the agents B, F and I, denoted as U_B, U_F and U_I are assumed to be exponential utility functions, since they have the particularity to be defined on $\mathbb{R}$. Thus, for any real-valued random variable X, which can be seen as the final wealth of a given agent (B, F or I)

$$\begin{aligned} U_B(X) &= -\exp(-\gamma_B X) \\ U_F(X) &= -\exp(-\gamma_F X) \\ U_I(X) &= -\exp(-\gamma_I X) \end{aligned}$$

where γ_B (resp. γ_F and γ_I) represents the risk-aversion coefficient of agent B (resp. agent F and agent I). These three parameters have to be positive. Note that, in this study, agent B is assumed to have the same risk aversion coefficient for both parts of the transaction. Or, in other words, the bank has the same utility function for both sides of the transaction. It is considered as an entity and not as two different parts, one per role. The study may be extended to the similar framework where the bank has a different risk aversion for its "insurer" role and its "issuer" one. Such a situation may illustrate the fact that two distinct departments of the bank could be involved in the global transaction. These parameters represent the sensibility of the agents towards risk and they have an impact on the utility criterion itself. They also are the coefficient of absolute risk aversion (exponential utility functions belong to the family of Constant Absolute Risk Aversion, or CARA, utility functions). Moreover, the particular choice of exponential utilities enables to play with the criterion according to the values taken by risk-aversion coefficients. On one hand, it is well-known that, when the risk aversion coefficient is small enough, maximising the expected utility is equivalent to a mean-variance criterion (assuming that the considered risk has a bounded variance). On the other, when γ is large, $\mathbb{E}[-\exp(-\gamma X)]$ is all the more important so since X is not "too much" negative. The attitude of the agent is not symmetric between possible gains or losses. This attitude appears quite logical: the agent does not want to bear "too much" important losses.

In order to simplify this study, only the current transaction is taken into account. It is equivalent to set the initial endowments of the agents to zero. Moreover, the transaction costs are assumed to be null. These assumptions may be dropped easily without modifying the global structure of the results. They enable us to derive simple expressions for the different parameters of the transaction.

3.2. *Characterization of the optimal insurance contract*

This subsection focuses on the problem of the relation between the firm, agent F, and the bank, agent B. It can easily be considered as an insurance relation as the firm calls on the bank to be hedged against the risk it faces. Indeed, to be protected, it pays a certain amount, π, and receives in exchange a compensation, $J(\theta)$, if loss $\Theta = \theta$ obtains. As to model this "insurance" relation between these agents, we use a classical insurance method, as that of Raviv [1979], which is described below:

Thus to model the relation between the bank and the firm, the standard assumptions of a passive insurer and of an insured maximising the expected utility of its final random wealth (Eq. (3)) under certain constraints are made. Hence, the insured determines the structure of the policy, which is optimal for it, whereas the insurer can only accept or refuse this new contract.

Hence, agent B is assumed to be passive and agent F to have the following optimization program, using previous notations

$$\begin{aligned} &\max_{\pi, J} \ \mathbb{E}\left[-\exp\{-\gamma_F(J(\Theta) - \Theta - \pi\beta_0)\}\right] \\ &\text{s.t.} \quad \mathbb{E}\left[-\exp\{-\gamma_B(\pi\beta_0 - J(\Theta))\}\right] \geq -1 \\ &\qquad\qquad 0 \leq J(\Theta) \leq \Theta \quad \mathbb{P}\ a.s. \end{aligned} \tag{$\mathcal{P}$}$$

where -1 corresponds to agent B's utility level if it does nothing and s.t. denotes "subject to". In particular, the constraint relative to $J(.)$ has been previously motivated (See Constraint (2)).

Agent F uses the decision criterion we previously described, whereas agent B only compares its expected utility of the final random wealth for two different situations: "insuring" agent F or doing nothing (See Eq. (4)).

The solving of such an optimization program will be the object of the next section.

Remark 2 (On the program $\mathcal{P}$)**:** The optimization program ($\mathcal{P}$) is standard in the insurance literature. Indeed, the first part of the transaction is a special case of Raviv's result, where there is no transaction cost and where utility functions are CARA. However, as the methodology will be useful to analyze the second part of the transaction, we present a full characterization of the optimal compensation.

Remark 3 (On the basis risk)**:** The results of this paper may be extended to the situation where there is a basis risk between the (individual) risk borne by the firm and the (global) risk covered by the insurance contract sold by the bank. Thus, if $\hat{\Theta}$ denotes the risk exposure of agent F and Θ the risk covered by the compensation paid by agent B, with $\hat{\Theta} \neq \Theta$ $\mathbb{P}$ a.s., the optimization program becomes

$$\begin{aligned}
&\max_{\pi, J} \; \mathbb{E}\left[-\exp\{-\gamma_F(J(\Theta) - \hat{\Theta} - \pi\beta_0)\}\right] \\
&\text{s.t.} \quad \mathbb{E}[-\exp\{-\gamma_B(\pi\beta_0 - J(\Theta))\}] \geq -1 \\
&\qquad\qquad 0 \leq J(\Theta) \leq \Theta \qquad \mathbb{P}\; a.s.
\end{aligned}$$

To come back to the framework of this paper, the "conditional certain equivalent" of the firm's exposure, with respect to the common risk Θ, is introduced

$$X(\Theta) \triangleq \frac{1}{\gamma_F} \ln \mathbb{E}\left[\exp(\gamma_F \hat{\Theta})/\Theta\right]$$

Hence, solving the optimization program with a basis risk is equivalent to solve the optimization program ($\mathcal{P}$), replacing the exposure Θ of agent F by $X(\Theta)$.

3.3. Design of the optimal weather bond

The structure of the bond is determined by agent B so that it is optimal for it, with respect to the utility criterion described in the previous section. The optimization variables are directly related to the structure of the financial contract: the coupon s, the amount α which is paid back when an event occurs and the price Φ. However, agent B is constrained by the existence of a counterpart. It is indeed necessary for the existence of the transaction that agent I has some interest in buying the weather bond. Its level of interest is given by a utility criterion and the investor compares it with that of a risk-free investment. Agent I is said to be "passive" as it can only decide to do or not the transaction.

Hence, to model the relation between the bank and the investor, the bank, agent B, is assumed to maximize the expected utility of its random final wealth (related to the global transaction and given by Eq. (5)) under certain constraints. On the other hand, the investor, agent I, only compares its expected utility of the final random wealth for two different situations: buying the bond from agent B or making a non-risky investment (see Eq. (6)).

This optimization program takes into account the first part of the transaction, i.e. the compensation function $J(\cdot)$ and the premium π. As both parts of the transaction are independent, conditionnally on agent B, the following optimization program is true for any couple $(J(\cdot), \pi)$, and in particular for $(J^*(\cdot), \pi^*)$. Hence, at the optimum, these quantities are logically the optimal ones.

$$\begin{aligned} &\max_{\Phi,\alpha,s}\ \mathbb{E}\left[-\exp\left\{-\gamma_B\left(\pi\beta_0 - N - J(\Theta) + \Phi\beta_0 - s\sum_{i=1}^{n}\beta_i + \frac{\alpha}{M}\Theta\right)\right\}\right] \\ &\text{s.t.}\quad \mathbb{E}\left[-\exp\left\{-\gamma_I\left(-\Phi\beta_0 + s\sum_{i=1}^{n}\beta_i + N - \frac{\alpha}{M}\Theta\right)\right\}\right] \geq -1 \end{aligned} \qquad (\bar{\mathcal{P}})$$

where -1 corresponds to agent I's utility level if it makes a non-risky investment and s.t. denotes "subject to".

Note that α and Λ form a system of parsimonious control variables. This second formulation of the optimization program will be favoured in the following in order to simplify the notations.

Note that the logic adopted for the financial investment in this article is not risk-neutral. Moreover, it is close to the framework of pricing via utility maximization as, for instance, in Hodges and Neuberger [1989] or in El Karoui and Rouge [2000]. Indeed, there is no underlying market where agents may build a replicating strategy for the bond. Moreover, the investor has a static point of view, it can only choose between the bond and cash. This logic is closer to that of insurance as the diversification potential of the bond for the investor's portfolio is not taken into consideration.

The solving of such an optimization program will be the object of the next section. However, note that, instead of solving the program $(\bar{\mathcal{P}})$ using standard variational control techniques, as in the following section, it is equivalent to directly introduce agent I's bound constraint into the minimization of agent B. This is possible as, given the constraint, a unique Λ is associated to any value of α. This method stresses the particular role played by both agents in the characterization of the bond: agent I determines the structure of the price or of Λ whereas agent B features the whole structure of the bond optimally, but using the price function given by agent I.

Moreover, both π and the compensation function $J(\cdot)$ play a very particular role in agent B's optimization program. Indeed, the bank determines the optimal structure of the bond, conditionally on the knowledge of $(\pi, J(\cdot))$. Therefore, in the very particular framework of exponential utilities, $(\pi, J(\cdot))$ may define a change of probability measures: the bank characterizes the optimal structure of the bond under a certain probability measure, which depends on both π and $J(\cdot)$. This comment underlines some potential extension of this study to more general relations between the bank and the firm.

4. Solving the optimization problems

The solving of the optimization problem for the global transaction leads to the solving of two optimization subproblems, as described in the previous section. For that reason, the first step concerns the relation between the firm and the bank, whereas the second one deals with the bond issue, i.e. the relation between agent B and agent I.

4.1. Solving the problem of the relation between the firm and the bank

As written in the previous section, the optimization program related to the relation between agent F and agent B is given by

$$\begin{aligned} &\min_{\pi,J}\ \mathbb{E}\left[\exp\{-\gamma_F(J(\Theta)-\Theta-\pi\beta_0)\}\right] \\ &\text{s.t.}\quad \mathbb{E}\left[\exp\{-\gamma_B(\pi\beta_0-J(\Theta))\}\right]\leq 1 \\ &\qquad\qquad 0\leq J(\Theta)\leq\Theta\quad \mathbb{P}\ a.s. \end{aligned}\tag{$\mathcal{P}$}$$

This program depends on two different parameters: π represents the "price" of the insurance contract, and J the compensation function, which gives the amount which is paid back when an event occurs.

4.1.1. Solving of the optimization program ($\mathcal{P}$). To solve ($\mathcal{P}$), variational control techniques are used, by introducing a positive Lagrange multiplier coefficient λ, which will be chosen optimally later. The modified global utility of the program is denoted as $\hat{U}$ and defined by

$$\hat{U}(\Theta, J(\Theta), \pi) = -\exp[-\gamma_F(J(\Theta)-\Theta-\pi\beta_0)] - \lambda\exp[-\gamma_B(\pi\beta_0 - J(\Theta))]$$

To solve ($\mathcal{P}$), we first solve ($\hat{\mathcal{P}}$) defined by

$$\max_{\pi,J}\mathbb{E}[\hat{U}(\Theta, J(\Theta), \pi)]\tag{$\hat{\mathcal{P}}$}$$

Then, so as to go back to ($\mathcal{P}$), the optimal value of the coefficient λ has to be determined, in such a way that the constraint is bound.

4.1.1.1. First-order conditions. Solving ($\hat{\mathcal{P}}$), in the particular framework of exponential utilities, leads to the following first order conditions at the optimum $(J^*(\cdot), \pi^*)$: $\forall\theta\in\mathcal{D}(\Theta)$,

– If $0 < J^*(\theta) < \theta$

$$\gamma_F\exp[-\gamma_F(J^*(\theta)-\theta-\pi^*\beta_0)] - \lambda\gamma_B\exp[-\gamma_B(\pi^*\beta_0 - J^*(\theta))] = 0\tag{7}$$

– If $J^*(\theta) = 0$

$$\gamma_F\exp[\gamma_F(\theta+\pi^*\beta_0)] - \lambda\gamma_B\exp[-\gamma_B(\pi^*\beta_0)] \leq 0\tag{8}$$

– If $J^*(\theta) = \theta$

$$\gamma_F \exp[\gamma_F(\pi^* \beta_0)] - \lambda \gamma_B \exp[-\gamma_B(\pi^* \beta_0 - \theta)] \geq 0 \tag{9}$$

On the one hand, these conditions are obtained, for any $\theta \in \mathcal{D}(\Theta)$ fixed, by partially deriving $\hat{U}$ with respect to $J(\theta)$ and taking its expected value at the optimum $(J^*(\cdot), \pi^*)$. As it has to be equal to zero, some conditions on their sign appear according to the value of $J^*(\theta)$, for any $\theta \in \mathcal{D}(\Theta)$. On the other hand, the partial derivative of $\hat{U}$ with respect to π is calculated and its expected value is taken at the optimum $(J^*(\cdot), \pi^*)$. It also has to be equal to zero.

The determination of these conditions and their sufficiency, as classical results in convex optimization, are given in appendixes.

4.1.1.2. Determination of two thresholds. The optimal level of compensation, $J^*(\theta)$, if risk of loss $\Theta = \theta$, $\theta \neq 0$, obtains, belong to the interval $[0, \theta]$, as Constraint (2) holds. In order to precise the structure of the compensation, it will be very useful to have some rules on θ giving whether $J^*(\theta) = 0$, $J^*(\theta) = \theta$ or $J^*(\theta) \in]0, \theta[$ (so that only one first-order condition would be valid). For that reason, $\theta^-(\pi^*, \lambda)$ and $\theta^+(\pi^*, \lambda)$, two intrinsic thresholds, may be introduced. They respectively denote the policy deductible level and the policy upper limit for a complete compensation i.e.

– $\theta^-(\pi^*, \lambda)$ is such that: if $J^*(\theta) = 0$ then

$$\gamma_F \exp[\gamma_F(\theta + \pi^* \beta_0)] - \lambda \gamma_B \exp[-\gamma_B(\pi^* \beta_0)]$$

is negative only if $\theta \leq \theta^-(\pi^*, \lambda)$.

– $\theta^+(\pi^*, \lambda)$ is such that: if $J^*(\theta) = \theta$ then

$$\gamma_F \exp[\gamma_F(\pi^* \beta_0)] - \lambda \gamma_B \exp[-\gamma_B(\pi^* \beta_0 - \theta)]$$

is positive only if $\theta \leq \theta^+(\pi^*, \lambda)$.

These thresholds may be calculated explicitly

$$\theta^-(\pi^*, \lambda) = -\frac{\gamma_B + \gamma_F}{\gamma_F}\pi^* \beta_0 + \frac{1}{\gamma_F} \ln\left(\frac{\lambda \gamma_B}{\gamma_F}\right)$$
$$\theta^+(\pi^*, \lambda) = \frac{\gamma_B + \gamma_F}{\gamma_B}\pi^* \beta_0 - \frac{1}{\gamma_B} \ln\left(\frac{\lambda \gamma_B}{\gamma_F}\right)$$

Hence, the following relation holds

$$\theta^+(\pi^*, \lambda) = -\frac{\gamma_F}{\gamma_B}\theta^-(\pi^*, \lambda)$$

and both thresholds have opposite signs. But $\mathcal{D}(\Theta) \subset \mathbb{R}_+$, both thresholds are positive. Hence

$$\theta^-(\pi^*, \lambda) = \theta^+(\pi^*, \lambda) = 0 \tag{10}$$

Thus, the optimal level of compensation, $J^*(\theta)$, if risk of loss $\Theta = \theta$, $\theta \neq 0$, obtains, always lies in the interval $]0, \theta[$, for any $\theta \in \mathcal{D}(\Theta)$, $\theta \neq 0$. Hence, boundaries are never reached and the framework of this study is more simple.

4.1.2. Structure of the optimal compensation J^* and of the optimal premium π^*. Now, we are in a position to write a relation between J^*, π^* and the other parameters for any values $\theta \in \mathcal{D}(\Theta), \theta \neq 0$. As $J^*(\theta) \in]0, \theta[$, the first-order condition (7) gives the following equation

$$\gamma_F \exp[-\gamma_F(J^*(\theta) - \theta - \pi^*\beta_0)] = \lambda\gamma_B \exp[-\gamma_B(\pi^*\beta_0 - J^*(\theta))]$$

and

$$J^*(\theta) = \frac{\gamma_F}{\gamma_B + \gamma_F}\theta + \pi^*\beta_0 - \frac{1}{\gamma_B + \gamma_F} \ln\left(\frac{\lambda\gamma_B}{\gamma_F}\right) \tag{11}$$

Moreover, the Eq. (10) gives the following relationship between π^* and λ

$$\pi^*\beta_0 = \frac{1}{\gamma_B + \gamma_F} \ln \frac{\lambda\gamma_B}{\gamma_F}$$

Hence, the optimal compensation is given for any values $\theta \in \mathcal{D}(\Theta)$, $\theta \neq 0$, by

$$J^*(\theta) = \frac{\gamma_F}{\gamma_B + \gamma_F}\theta$$

But this result can be extended to $\theta = 0$ as $J^*(0) = 0$ by using constraint (2). Finally

$$J^*(\theta) = \frac{\gamma_F}{\gamma_B + \gamma_F}\theta \qquad \forall\, \theta \in \mathcal{D}(\Theta)$$

The optimal pricing rule for the insurance conttract is obtained by binding the constraint at the optimum. Hence

$$\pi^*\beta_0 = \frac{1}{\gamma_B} \ln \mathbb{E}\left[\exp(\gamma_B J^*(\Theta))\right]$$

These results may be summarised in the following proposition:

Proposition 1: *The optimal level of premium*, π^*, *is given by*

$$\pi^*\beta_0 = \frac{1}{\gamma_B} \ln \mathbb{E}\left[\exp(\gamma_B J^*(\Theta))\right] \tag{12}$$

And the optimal compensation, $J^*(\theta)$, *when a risk of loss* $\theta \in \mathcal{D}(\Theta)$ *obtains*, *is given by*

$$J^*(\theta) = \frac{\gamma_F}{\gamma_B + \gamma_F}\theta \quad \forall\, \theta \in \mathcal{D}(\Theta) \tag{13}$$

The optimal structure of the compensation is coherent with the Borch's theorem (see for instance, Eeckhoudt and Gollier [1995]) concerning risk sharing and mutualisation. J^* is indeed proportional to the obtained loss. The proportional coefficient is the ratio of agent F's risk aversion coefficient with respect to the sum of both agent F's and agent B's coefficients. It may be seen as agent F's relative risk aversion: i.e. if both agents have the same attitude towards risk, they will perfectly share the obtained loss; but the more risk averse agent F is, relatively to agent B, the larger is the compensation. This result does not depend on a particular a priori given form of the compensation. Indeed, it is optimal among all the possible compensation structures, not only among the proportional one. Moreover, the optimal π^* is a non-linear function of the compensation. This non-linearity is one of the major aspects of the transaction, as we shall see in the section dedicated to the study of the bond emission.

Remark 4 (On the optimal compensation)**:** The shape of the optimal compensation does not include a deductible. It comes directly from the assumption of no transaction cost.

4.2. *Solving the problem of the relation between the bank and the investor*

The optimization program related to the relation between agent B and agent I is given by

$$\begin{aligned} &\min_{\Lambda,\alpha}\ \mathbb{E}\left[\exp\left\{-\gamma_B\left(\pi\beta_0 - J(\Theta) - \Lambda + \frac{\alpha}{M}\Theta\right)\right\}\right] \\ &\text{s.t.}\quad \mathbb{E}\left[\exp\left\{-\gamma_I\left(\Lambda - \frac{\alpha}{M}\Theta\right)\right\}\right] \leq 1 \end{aligned} \qquad (\bar{\mathcal{P}})$$

This program depends on two different parameters: $\Lambda = -\Phi\beta_0 + s\sum_{i=1}^{n}\beta_i + N$ represents the capitalized amount of cash which is received by the investor, and α the amount which is paid back when an event occurs.

As written in the previous section, the optimal structure of the bond is determined by agent B, which knows the price function determined by agent I. In other words, agent I determines the structure of Λ, whereas agent B focuses on the key variable α.

4.2.1. Optimal characterization of the weather bond. This finite dimensional problem is a special case of the previous problem. Using the previous results, we know that the optimal solution is proportional. Hence, looking for the optimal value of α (solution among the proportional functions) is equivalent to look for the general optimal solution. Moreover, this problem is much simpler than the previous one, as there is no constraint imposed on α. Consequently, solving the optimization program $(\bar{\mathcal{P}})$ is immediate and leads to the following proposition

Proposition 2: *The price function*, Φ^*, *of the weather bond is given by the following formula*

$$\Phi^*\beta_0 = s^*\sum_{i=1}^{n}\beta_i + N - \frac{1}{\gamma_I}\ln\mathbb{E}\left[\exp\left(\gamma_I\frac{\alpha^*}{M}\Theta\right)\right] \qquad (14)$$

The optimal amount which is paid back when an event occurs is given by

$$\alpha^* = \frac{\gamma_B \gamma_F}{(\gamma_B + \gamma_F)(\gamma_B + \gamma_I)} M \tag{15}$$

This constraint leads to the characterization of a unique price function of the bond: agent I imposes a certain relationship between the variables s, Φ and α (or, using the simplifying notation, between Λ and α). The uniqueness of the price function comes from the fact that, for any α, it is possible to find a unique Λ, which binds agent I's constraint, as written below. Agent B takes this relationship into account to determine the optimal structure of the bond. In other words, agent I determines the structure of Λ, whereas agent B focuses on the key variable α, which conditions the risky part of the bond. For these reasons, we can interpret the bond emission as the signing of a "minimal" contract, i.e. the bond characteristics we obtain will be "minimum" in two ways as they represent both a threshold of interest for the investor and a threshold of hedging for the bank.

4.2.1.1. Some comments on the pricing rule. The right-hand side of the pricing rule (14) represents the amount that agent I is willing to pay for the bond characterized by (s^*, α^*). The price function is non-linear, far from the standard logic of pricing involving expected value and linearity. Even if with exponential utility functions, initial endowments do not play any role in the results, here, there is a non-constant dependence on the risky flows of the bond, due to this non-linearity. Note that binding agent I's constraint is not trivial, since it has an impact on the whole structuration of the bond, by introducing this non-linear aspect in the price function.

Moreover, Φ^* is a very interesting price for a marketing point of view, as it is obviously lower than the historical price denoted as P and defined as the expected value of the sum of all discounted flows related to the bond with respect to the historical probability measure $\mathbb{P}$. Therefore, the investor has to pay less than the historical expected value of the discounted bond flows to buy the bond. Indeed, when comparing Φ^* and P, we obtain $\Phi^* < P$ as function exp is convex

$$\mathbb{E}\left[\exp\left(\gamma_I \frac{\alpha^*}{M}\Theta\right)\right] > \exp \mathbb{E}\left(\gamma_I \frac{\alpha^*}{M}\Theta\right)$$

$$\Phi^* \beta_0 < s^* \sum_{i=1}^{n} \beta_i + N - \frac{\alpha^*}{M}\mathbb{E}(\Theta) = P\beta_0$$

4.2.1.2. Some comments on the bond structure. The optimal amount which is paid back when an event occurs, α^*, is proportional. It is optimal among all possible functions and does not require any particular restriction on the optimisation set. Consequently, it does not depend on a particular a priori given structure of payments. On the other hand, note that the coupon level s^* of the bond appears clearly as a marketing tool to appeal the investor. Indeed, the price function Φ^* of the bond depends linearly on the coupon level. The spread between α^*, the amount which is paid back when an event occurs, and s^* is simply transferred to the bond price. There is no unique determination of the optimal coupon level s^*. The transfer problem between Φ^* and s^* leads to a situation where there is an infinity of solutions.

The optimum is defined by (Φ^*, s, α^*) where Φ^* and α^* are optimally chosen. Then, the "optimal" value of s is determined by the investor according to the structure of its current portfolio, and the diversification dimension of the weather bond for the investor can be finally taken into account. Hence, considering only the global variable Λ is completely equivalent to considering both s and Φ.

This proposition shows an important result for the structuration of the bond: not only the pricing of the product but also its characteristics and stakes are determined here. In this framework, α^* play a huge role as being the keystone of the bond structure.

Remark 5: Replacing α^* by its value, the price function, Φ^*, of the weather bond may be now written as

$$\Phi^* \beta_0 = s^* \sum_{i=1}^{n} \beta_i + N - \frac{1}{\gamma_I} \ln \mathbb{E}\left[\exp(\Gamma\Theta)\right]$$

where

$$\Gamma = \frac{\gamma_B \gamma_F \gamma_I}{(\gamma_B + \gamma_F)(\gamma_B + \gamma_I)}$$

is a function of all the agents' risk aversion coefficient.
In the particular case when all agents are not very risk averse (Γ small enough), we obtain, with a Taylor expansion with order 2 in the neighbourhood of 0

$$\Phi^* \beta_0 \sim s^* \sum_{i=1}^{n} \beta_i + N - \frac{\gamma_B \gamma_F}{(\gamma_B + \gamma_F)(\gamma_B + \gamma_I)} \mathbb{E}(\Theta) \\ -\frac{1}{2}\gamma_I \left(\frac{\gamma_B \gamma_F}{(\gamma_B + \gamma_F)(\gamma_B + \gamma_I)} \right)^2 \mathbb{E}(\Theta^2)$$

It can be seen as the sum of the historical expected value of the discounted bond's flows and a "premium" term which is all the more negative so since agent I is risk averse. We find again the noteworthy property $\Phi^* < P$.

4.2.1.3. Different interpretations of the bond structure. The degree of freedom in the choice of s^* leads to different possible interpretation of the bond structure, which can be used as different marketing strategies to appeal the investor.

- First, if s^* is taken greater than α^*—this is probably the most natural situation, when the amount α^* which is paid back if an event occurs, is smaller than the coupon s^*, the product has a real bond structure: all the net cash flows related to the bond are positive for agent I (apart from the price, of course).
- Secondly, if s^* is taken so that the bond is issued at par, i.e. $\Phi^* = N$, the structure of the bond is very classical and s^* is given by

$$s^* \sum_{i=1}^{n} \beta_i = N(\beta_0 - 1) + \frac{1}{\gamma_I} \ln \mathbb{E}\left[\exp(\Gamma\Theta)\right]$$

In the particular case when all agents are not very risk averse (Γ small enough), we obtain, with a Taylor expansion with order 2 in the neighbourhood of 0

$$s^* \sim \frac{\frac{\alpha^*}{M}\mathbb{E}(\Theta) - \frac{1}{2}\gamma_I\left(\frac{\alpha^*}{M}\right)^2\mathbb{E}(\Theta^2) + N(\beta_0 - 1)}{\sum_{i=1}^n \beta_i}$$

where α^* is given by Eq. (15).

– Finally, if $\Phi^*\beta_0 = N$, the bond is reduced to a yearly exchange of flows, conditionally on the occurrence of an event. Hence, each year, agent I will systematically receive s^* whereas she will pay α^* to agent B if and only if an event occurs. This is quite similar to a swap structure. Moreover, note that s^* is given by

$$s^* \sum_{i=1}^n \beta_i = \frac{1}{\gamma_I} \ln \mathbb{E}\left[\exp(\Gamma\Theta)\right]$$

In the particular case when all agents are not very risk averse (Γ small enough), we obtain, with a Taylor expansion with order 2 in the neighbourhood of 0

$$s^* \sim \frac{\frac{\alpha^*}{M}\mathbb{E}(\Theta) - \frac{1}{2}\gamma_I\left(\frac{\alpha^*}{M}\right)^2\mathbb{E}(\Theta^2)}{\sum_{i=1}^n \beta_i}$$

where α^* is given by Eq. (15).

Note that the interpretation in terms of a swap is also valid for any price Φ^*. Indeed, as

$$\Phi^*\beta_0 - N = s^* \sum_{i=1}^n \beta_i - \frac{1}{\gamma_I} \ln \mathbb{E}\left[\exp\left(\gamma_I \frac{\alpha^*}{M}\Theta\right)\right]$$

The first term can be seen as a front fee whereas the second represents the swapped flows. The front fee enables the swap initial value to be set to 0.

4.2.2. Risk valuation of both agents: An entropy approach. The pricing rule of the weather bond (14) has a non-linear component with respect to the risk Θ

$$-\frac{1}{\gamma_I} \ln \mathbb{E}\left[\exp\left(\gamma_I \frac{\alpha^*}{M}\Theta\right)\right]$$

Thanks to the entropy[3] approach, another interpretation may be obtained for this quantity. Indeed, considering a new probability measure $\mathbb{Q}$, absolutely continuous with respect to $\mathbb{P}$

$$\begin{aligned}
-\frac{1}{\gamma_I} \ln \mathbb{E}\left[\exp\left(\gamma_I \frac{\alpha^*}{M}\Theta\right)\right] &= -\frac{1}{\gamma_I} \ln \mathbb{E}_{\mathbb{Q}}\left(\frac{\exp\left(\gamma_I \frac{\alpha^*}{M}\Theta\right)}{\frac{d\mathbb{Q}}{d\mathbb{P}}}\right) \\
&\geq -\frac{1}{\gamma_I}\mathbb{E}_{\mathbb{Q}}\left(\gamma_I \frac{\alpha^*}{M}\Theta - \ln \frac{d\mathbb{Q}}{d\mathbb{P}}\right) \\
&= -\mathbb{E}_{\mathbb{Q}}\left(\frac{\alpha^*}{M}\Theta\right) + \underbrace{\frac{1}{\gamma_I}\mathbb{E}\left[\frac{d\mathbb{Q}}{d\mathbb{P}} \ln\left(\frac{d\mathbb{Q}}{d\mathbb{P}}\right)\right]}_{\frac{1}{\gamma_I} h(\mathbb{Q}/\mathbb{P})}
\end{aligned}$$

Moreover, as Θ is bounded,[4] a probability measure, $\mathbb{Q}_I$, may be defined as

$$\frac{d\mathbb{Q}_I}{d\mathbb{P}} = \frac{\exp\left(\gamma_I \frac{\alpha^*}{M}\Theta\right)}{\mathbb{E}\left[\exp\left(\gamma_I \frac{\alpha^*}{M}\Theta\right)\right]}$$

It is optimal for agent I using the entropic criterion in the sense that

$$-\frac{1}{\gamma_I}\ln\mathbb{E}\left[\exp\left(\gamma_I\frac{\alpha^*}{M}\Theta\right)\right] = -\mathbb{E}_{\mathbb{Q}_I}\left(\frac{\alpha^*}{M}\Theta\right) + \frac{1}{\gamma_I}\mathbb{E}\left[\frac{d\mathbb{Q}_I}{d\mathbb{P}}\ln\left(\frac{d\mathbb{Q}_I}{d\mathbb{P}}\right)\right]$$

$\mathbb{Q}_I$ is different from a risk-neutral measure. It does not play any role in the valuation of the bond's parameters. But it is an interpretation tool for the assessment of risk by agent I, as its density depends on the risk aversion coefficient of the investor and on the random part of its capitalized flows.

Using the same arguments, it is possible to valuate the Θ -exposure of agent B

$$-\frac{1}{\gamma_B}\ln\mathbb{E}\left[\exp\left(-\gamma_B\left(-J^*(\Theta)+\frac{\alpha^*}{M}\Theta\right)\right)\right]$$

Thus, the probability measure $\mathbb{Q}_B$, defined by the following Radon-Nikodym density

$$\frac{d\mathbb{Q}_B}{d\mathbb{P}} = \frac{\exp\left[-\gamma_B\left(-J^*(\Theta)+\frac{\alpha^*}{M}\Theta\right)\right]}{\mathbb{E}\left[\exp\left\{-\gamma_B\left(-J^*(\Theta)+\frac{\alpha^*}{M}\Theta\right)\right\}\right]}$$

may be introduced. $\mathbb{Q}_B$ also corresponds to the probability which minimizes the relative entropy

$$\begin{aligned} &-\frac{1}{\gamma_B}\ln\mathbb{E}\left[\exp\left(-\gamma_B\left(-J^*(\Theta)+\frac{\alpha^*}{M}\Theta\right)\right)\right] \\ &\quad = -\mathbb{E}_{\mathbb{Q}_B}\left(-J^*(\Theta)+\frac{\alpha^*}{M}\Theta\right) + \frac{1}{\gamma_B}\mathbb{E}\left[\frac{d\mathbb{Q}_B}{d\mathbb{P}}\ln\left(\frac{d\mathbb{Q}_B}{d\mathbb{P}}\right)\right] \end{aligned}$$

Moreover, by simply replacing α^* and J^* by their respective value given by (15) and (13), we directly obtain

$$\frac{d\mathbb{Q}_B}{d\mathbb{P}} = \frac{d\mathbb{Q}_I}{d\mathbb{P}}$$

and the following result

Proposition 3: *When doing the weather bond transaction, both agents have the same view on the risk Θ since*

$$\mathbb{Q}_B = \mathbb{Q}_I \tag{16}$$

Hence, their valuation of risk is composed of two terms, which are identical for both of them: the first one correspond to a risk-neutral price whereas the second one is a penalisation. The difference between both agents' valuation simply lies in the weighting of the penalisation since this weight depends on their risk aversion coefficient.

4.3. Some additional comments

4.3.1. Some remarks on the bank portfolio. As described in the general introduction and in the different sections of this paper, the flows, related to the transaction and capitalized from the moment when they take place to year n, can be written for agent B as follows

$$\mathbf{V}_B = \pi^* \beta_0 - J^*(\Theta) + \Phi^* \beta_0 - s \sum_{i=1}^{n} \beta_i - N + \frac{\alpha^*}{M} \Theta$$

Replacing the different parameters by the optimal values, obtained when solving the different optimization programs involving the three agents, the random value of the bank portfolio, denoted as $\mathbf{V}_B$, is given, at the end of the considered period, by the $\mathbb{P}\ a.s.$ following equation

$$\begin{aligned}\mathbf{V}_B = {} & \frac{\gamma_F \gamma_I}{(\gamma_B + \gamma_F)(\gamma_B + \gamma_I)} \Theta \\ & + \frac{1}{\gamma_B} \ln \mathbb{E}\left[\exp\left(\frac{\gamma_B \gamma_F}{\gamma_B + \gamma_F} \Theta\right)\right] - \frac{1}{\gamma_I} \ln \mathbb{E}\left[\exp\left(\frac{\gamma_I \gamma_B \gamma_F}{(\gamma_B + \gamma_F)(\gamma_B + \gamma_I)} \Theta\right)\right]\end{aligned} \tag{17}$$

Agent B is not completely hedged against the risk Θ and against the risky flows of the compensation it will have to pay when an event occurs. In this sense, the bank has some speculative behaviour.

Its remaining exposition is proportional to the "relative" risk aversion of both agent I and agent F: i.e. respectively $\frac{\gamma_I}{(\gamma_B + \gamma_I)}$ and $\frac{\gamma_F}{(\gamma_B + \gamma_F)}$. The amounts agent B has received for both risky parts of the transaction correspond to the second line of the formula. Both of them are non-linear functions of Θ. In the particular case when all agents are not very risk averse (their respective risk-aversion coefficients are small enough), we obtain, with a Taylor expansion with order 1 in the neighbourhood of 0 ($\mathbb{P}\ a.s.$ relation)

$$\mathbf{V}_B \sim \frac{\gamma_F \gamma_I}{(\gamma_B + \gamma_F)(\gamma_B + \gamma_I)} \Theta + \frac{\gamma_F}{\gamma_B + \gamma_F} \mathbb{E}(\Theta) - \frac{\gamma_B \gamma_F}{(\gamma_B + \gamma_F)(\gamma_B + \gamma_I)} \mathbb{E}(\Theta)$$

or

$$\mathbf{V}_B \sim \frac{\gamma_F \gamma_I}{(\gamma_B + \gamma_F)(\gamma_B + \gamma_I)} (\Theta - \mathbb{E}(\Theta))$$

At the first order, the exposition of the bank is limited to the variations of Θ around its historical expected value. A Taylor expansion with order 2 in the neighbourhood of 0 adds a negative term which depends on the variance of Θ ($\mathbb{P}$ $a.s.$ relation)

$$\begin{aligned}\mathbf{V}_B \sim\ & \frac{\gamma_F \gamma_I}{(\gamma_B+\gamma_F)(\gamma_B+\gamma_I)}(\Theta - \mathbb{E}(\Theta)) \\ & -\frac{1}{2}\left[\left(\frac{\gamma_F}{(\gamma_B+\gamma_F)(\gamma_B+\gamma_I)}\right)^2 \left[\gamma_B^3 + \gamma_I^2\gamma_B + 3\gamma_I\gamma_B^2\right]\right]\mathbb{E}(\Theta^2)\end{aligned}$$

4.3.2. Some remarks on the aversion coefficients

4.3.2.1. An asymptotic study of the results. An asymptotic study, when one of the aversion coefficient tends to the infinity or to zero, of the main results of this paper leads to some natural intepretation:

- Asymptotics when γ is infinite
  - If γ_B tends to the infinity (i.e. the bank is infinitely risk-averse)

    $$J^*(\Theta) \to 0 \quad \mathbb{P}\ a.s.$$
    $$\pi^* \to 0$$

    There is no transaction between the bank and the firm. The bank does not want to hedge the firm's risk. In that case, no weather bond can be issued.
  - If γ_F tends to the infinity (i.e. the firm is infinitely risk-averse)

    $$J^*(\Theta) \to \Theta \quad \mathbb{P}\ a.s.$$
    $$\pi^* \to \frac{1}{\beta_0\gamma_B}\ln\mathbb{E}\left[\exp(\gamma_B\Theta)\right]$$

    The firm wants to be covered totally. In this particular case, a weather bond may be issued with the following characteristics

    $$\alpha^* \to \frac{\gamma_B}{\gamma_B+\gamma_I}M$$
    $$\Phi^*\beta_0 \to s\sum_{i=1}^{n}\beta_i + N - \frac{1}{\gamma_I}\ln\mathbb{E}\left[\exp\left(\frac{\gamma_B\gamma_I}{\gamma_B+\gamma_I}\Theta\right)\right]$$

    The structure is comparable with that of an insurance relation between the bank and the investor on the risk Θ.
  - If γ_I tends to the infinity (i.e. the investor is infinitely risk-averse).

    $$\alpha^* \to 0$$

    No weather bond can be issued.
- Asymptotics when γ is null
  - If γ_B tends to 0 (i.e. the bank is risk-neutral in the utility sense)

    $$J^*(\Theta) \to \Theta \qquad \mathbb{P}\ a.s.$$

    The bank accepts to cover the whole risk Θ. No weather bond can be issued as it keeps the risk in its portfolio

    $$\alpha^* \to 0$$

– If γ_F tends to 0 (i.e. the firm is risk-neutral in the utility sense)

$$J^*(\Theta) \to 0 \quad \mathbb{P}\ a.s.$$

The firm does not want to hedge its risk Θ. No transaction will take place.

– If γ_I tends to zero (i.e. the investor is risk-neutral in the utility sense). A weather bond may be issued with the following characteristics

$$\alpha^* \to \frac{\gamma_F}{\gamma_B + \gamma_F} M$$

and the non-linear "price of risk"

$$-\frac{1}{\gamma_I} \ln \mathbb{E}\left[\exp\left(\gamma_I \frac{\alpha^*}{M} \Theta\right)\right] \to 0$$

4.3.2.2. Behaviour of the non-linear "price of risk" related to the weather bond. A simple study of the monotonicity of the non-linear "price of risk", i.e.

$$\begin{aligned} PR(\gamma_B, \gamma_F, \gamma_I) &\triangleq -\frac{1}{\gamma_I} \ln \mathbb{E}\left[\exp\left(\gamma_I \frac{\alpha^*}{M} \Theta\right)\right] \\ &= -\frac{1}{\gamma_I} \ln \mathbb{E}\left[\exp\left(\frac{\gamma_I \gamma_B \gamma_F}{(\gamma_B + \gamma_F)(\gamma_B + \gamma_I)} \Theta\right)\right] \end{aligned}$$

with respect to the different risk aversion coefficients leads to the following conclusions

– *With respect to γ_B*: α^* is a strictly increasing function of γ_B till the level $\sqrt{\gamma_I \gamma_F}$ and a strictly decreasing function of γ_B after this threshold. Consequently, as *PR* is a decreasing function of α^* (in our framework where Θ takes only positive values), *PR* is a decreasing function of γ_B till the level $\sqrt{\gamma_I \gamma_F}$ and an increasing function of γ_B after this threshold.
– *With respect to γ_F*: α^* is a strictly decreasing function of γ_F till the level $\gamma_B - 1$ and a strictly increasing function of γ_F after this threshold. Consequently, as *PR* is a decreasing function of α^*, *PR* is an increasing function of γ_F till the level $\gamma_B - 1$ and a decreasing function of γ_F after this threshold.
– *With respect to γ_I*: According to the previous interpretation of the probability measure $\mathbb{Q}_I$ as the minimal relative entropy measure, the non-linear "price of risk" may be written as

$$PR(\gamma_B, \gamma_F, \gamma_I) = \sup_{\mathbb{Q} \ll \mathbb{P}} \left\{ -\mathbb{E}_{\mathbb{Q}}\left(\frac{\gamma_B \gamma_F}{(\gamma_B + \gamma_F)(\gamma_B + \gamma_I)} \Theta\right) + \frac{1}{\gamma_I} h(\mathbb{Q}/\mathbb{P}) \right\}$$

However, the relative entropy of any absolutely continuous measure $\mathbb{Q}$ with respect to $\mathbb{P}$ is always negative and as Θ takes only positive values, the expected value is positive. Morevoer, as $\frac{\alpha^*}{\gamma_I}$ is a strictly decreasing function of γ_I, this expected value is a decreasing function of γ_I. Hence, *PR* is an increasing function of γ_I.

4.3.2.3. A VaR interpretation of the aversion coefficients. A standard question when using utility functions is that of quantifying the different risk aversion coefficients. Several criteria may be used to characterise these parameters. Among them, the VaR criterion (Value at Risk) could help the bank (for example) to choose its aversion coefficient γ_B. The VaR criterion is simply defined as the maximum amount $-A$ ($A < 0$) an agent is ready to lose for a given probability level δ. This limit is imposed on the terminal value $\mathbf{V}$ of her portfolio

$$\mathbb{P}(\mathbf{V} < A) \leq \delta \quad \text{or} \quad \mathbb{P}(-\mathbf{V} \geq -A) \leq \delta$$

For a given portfolio, A is a function of δ, corresponding to the δ-quantile, also called VaR.

In the particular framework of this study, assuming that the bank imposes a VaR criterion on the terminal value of its portfolio, $\mathbf{V}_B$, and given its risk aversion coefficient γ_B, it is possible to link together the VaR and the δ-quantile of the Θ-distribution. Indeed, using the expression of $\mathbf{V}_B$ obtained in (17), which may simply be rewritten as

$$\mathbf{V}_B = \varphi(\gamma_B)\Theta + \psi(\gamma_B)$$

where φ and ψ are two deterministic functions of γ_B , we obtain

$$\mathbb{P}(\mathbf{V}_B < A) = \mathbb{P}\left(\Theta < \frac{A - \psi(\gamma_B)}{\varphi(\gamma_B)}\right) \leq \delta$$

Hence, knowing the distribution function of Θ leads to the characterisation of γ_B using the δ-quantile, q_δ^Θ, since

$$\frac{A - \psi(\gamma_B)}{\varphi(\gamma_B)} \simeq q_\delta^\Theta \quad \text{(with } = \text{ if the } \Theta \text{ -distribution is continuous)}$$

Note that q_δ^Θ only depends on the Θ-distribution and on δ. In particular, γ_B does not influence this quantile.

Consequently, if the bank has an idea of its VaR, then it can determine a suitable value for its risk aversion parameter γ_B.

5. Concluding remarks

The main contribution of our study is to provide a way of completely characterizing a bond whose coupons depend on the occurrence of a weather event. One particularity of our analysis is to explore this transaction as a whole: from the firm which needs a hedge against weather risk to small bondholders. In this framework, given some basic assumptions about the involved agents, we are able to derive simultaneously both the bond price and the amount which is paid back when an event occurs, in a simple fashion. On the other hand, we adopt a static point of view, far from the standard risk-neutral logic, as there is no underlying market. The pricing of the product is not as important as its structuration. Thus, this study can be extended to more general structured transactions involving several agents and can go far beyond the simple frame of weather derivatives. For example, it can also be useful for bonds and even swaps, whose payments depend on the occurrence of a particular event (weather, catastrophe, credit), as no particular assumptions are required in this article regarding the considered events ε_i.

The study developed in this paper is robust to changes in many assumptions. Examples of possible generalisations which do not alter the basic arguments are allowing weather events to be correlated, permitting the potential loss amount M to be random, introducing transaction costs for agent B, or initial endowments and portfolios for the different agents, especially for the investor, so that the diversification potential of the weather bond be fully taken into account.

One assumption to which the results may be sensitive is the basic supposition that agents have exponential utilities. This enables us to derive nice expressions for the different parameters of the transaction. However, the extend to which the use of different utility functions, especially power utilities, may modify the results, is actually the subject of further investigation.

Moreover, as discussed before, the methodology we develop in this paper may certainly be useful for the analysis of different types of structured transaction. In particular, allowing the bank to have a strategic behaviour by taking into account its hedging strategy when insuring the firm, or more generally, allowing agents to have more interaction between them, may be an interesting generalisation of this model. This is also a field of research we currently explore.

Appendixes: Proof of Proposition 1

A.1. Optimality in J

Let ε be a real number in [0, 1]. From now on, we denote as J^* the optimal compensation, as π^* the optimal premium and as J any function satisfying the constraint

$$0 \leq J(\theta) \leq \theta \quad \forall \theta \in \mathcal{D}(\Theta)$$

Then, if for any $\theta \in \mathcal{D}(\Theta)$, we define

$$J^{\varepsilon}(\theta) = (1-\varepsilon)J^*(\theta) + \varepsilon J(\theta) = J^*(\theta) + \varepsilon(J(\theta) - J^*(\theta))$$

If

$$\hat{U}(\Theta, J^*(\Theta), \pi^*) = -\exp[-\gamma_F(J^*(\Theta) - \Theta - \pi^*\beta_0)] - \lambda \exp[-\gamma_B(\pi^*\beta_0 - J^*(\Theta))]$$

We have

$$\mathbb{E}[\hat{U}(\Theta, J^*(\Theta), \pi^*)] \geq \mathbb{E}[\hat{U}(\Theta, J^{\varepsilon}(\Theta), \pi^*)]$$

or, using the concavity of $\hat{U}$

$$\mathbb{E}\left[\frac{\partial \hat{U}}{\partial J}(J^*(\Theta)) \times (J(\Theta) - J^*(\Theta))\right] \leq 0$$

with

$$\frac{\partial \hat{U}}{\partial J}(J^*(\Theta)) = \gamma_F \exp[-\gamma_F(J^*(\Theta) - \Theta - \pi^*\beta_0)] - \lambda\gamma_B \exp[-\gamma_B(\pi^*\beta_0 - J^*(\Theta))]$$

Hence

$$\begin{aligned}
&\mathbb{E}\left[\mathbf{1}_{(0<J^*(\Theta)<\Theta)}\frac{\partial \hat{U}}{\partial J}(J^*(\Theta)) \times (J(\Theta) - J^*(\Theta))\right] \\
&\quad + \mathbb{E}\left[\mathbf{1}_{(J^*(\Theta)\,=\,0)}\frac{\partial \hat{U}}{\partial J}(J^*(\Theta)) \times J(\Theta)\right] \\
&\quad + \mathbb{E}\left[\mathbf{1}_{(J^*(\Theta)\,=\,\Theta)}\frac{\partial \hat{U}}{\partial J}(J^*(\Theta)) \times (J(\Theta) - \Theta)\right] \\
&\leq 0
\end{aligned}$$

Consequently, we need to study the sign of $\frac{\partial \hat{U}}{\partial J}(J^*(\Theta))$ on three disjoint sets:

- On the set $\{0 < J^*(\Theta) < \Theta\}$, we can choose enough functions J to make the sign of $J(\Theta) - J^*(\Theta)$ vary as much as we wish. We can also choose J so that, on the boundaries, when $J^*(\Theta) = 0$, $\mathbb{P}\ a.s.$, $J(\Theta) = 0$, $\mathbb{P}\ a.s.$ and when $J^*(\Theta) = \Theta$, $\mathbb{P}\ a.s.$, $J(\Theta) = \Theta$, $\mathbb{P}\ a.s.$. Thus we deduce that, on this set, the derivative is null on average and this can be satisfied only if

$$\frac{\partial \hat{U}}{\partial J}(J^*(\Theta)) = 0 \quad \mathbb{P}\ a.s.$$

- On the set $\{J^*(\Theta) = 0\}$, as $J(\Theta)$ is positive $\mathbb{P}\ a.s.$ by assumption, we deduce that, on this set, the derivative is negative on average and this can be satisfied only if

$$\frac{\partial \hat{U}}{\partial J}(J^*(\Theta)) \leq 0 \quad \mathbb{P}\ a.s.$$

- Finally, on the set $\{J^*(\Theta) = \Theta\}$, as $(J(\Theta) - \Theta)$ is negative $\mathbb{P}\ a.s.$ by assumption, we deduce that, on this set, the derivative is positive on average and this can be satisfied only if

$$\frac{\partial \hat{U}}{\partial J}(J^*(\Theta)) \geq 0 \quad \mathbb{P}\ a.s.$$

A.2. Optimality in π

Let ε a real number in [0, 1]. From now on, we denote as π^* the optimal premium, as J^* the optimal compensation and as π any other function.

Then, if for any $\theta \in \mathcal{D}(\Theta)$, we define

$$\pi^\varepsilon = (1-\varepsilon) \times \pi^* + \varepsilon \times \pi = \pi^* + \varepsilon(\pi - \pi^*)$$

we have

$$\mathbb{E}\,[\hat{U}(\Theta, J^*(\Theta), \pi^*)] \geq \mathbb{E}\,[\hat{U}(\Theta, J^*(\Theta), \pi^\varepsilon)]$$

or

$$\mathbb{E}\left[\frac{\partial \hat{U}}{\partial \pi}(\pi^*) \times (\pi - \pi^*)\right] \leq 0$$

with

$$\frac{\partial \hat{U}}{\partial \pi}(\pi^*) = \gamma_F \beta_0 \exp[-\gamma_F(J^*(\Theta) - \Theta - \pi^* \beta_0)] - \lambda \gamma_B \beta_0 \exp[-\gamma_B(\pi^* \beta_0 - J^*(\Theta))]$$

Thus we can choose enough functions π to make the sign of $\pi - \pi^*$ vary. So, we deduce that the derivative is null on average

$$\mathbb{E}\left[\frac{\partial \hat{U}}{\partial \pi}(\pi^*)\right] = 0$$

A.3. Particular case of exponential utilities

In the particular framework of this article, i.e. exponential utilities, the conditions, previously obtained, can be written as:

– *The first order conditions* are:
  If $0 < J^*(\Theta) < \Theta$, $\mathbb{P}\ a.s.$, then

$$\gamma_F \exp[-\gamma_F(J^*(\Theta) - \Theta - \pi^* \beta_0)] - \lambda \gamma_B \exp[-\gamma_B(\pi^* \beta_0 - J^*(\Theta))] = 0 \quad \mathbb{P}\ a.s.$$

  and

$$\mathbb{E}\left[\begin{array}{c} \gamma_F \exp[-\gamma_F(J^*(\Theta) - \Theta - \pi^* \beta_0)] \\ -\lambda \gamma_B \exp[-\gamma_B(\pi^* \beta_0 - J^*(\Theta))] \end{array}\right] = 0$$

– *The boundary conditions* are:
  If $J^*(\Theta) = 0$, $\mathbb{P}\ a.s.$, then

$$\gamma_F \exp[\gamma_F(\Theta + \pi^* \beta_0)] - \lambda \gamma_B \exp[-\gamma_B(\pi^* \beta_0)] \leq 0 \quad \mathbb{P}\ a.s.$$

If $J^*(\Theta) = \Theta$, $\mathbb{P}$ $a.s.$, then

$$\gamma_F \exp[\gamma_F(\pi^*\beta_0)] - \lambda\gamma_B \exp[-\gamma_B(\pi^*\beta_0 - \Theta)] \geq 0 \quad \mathbb{P}\ a.s.$$

We can notice that the second first order condition is redundant with the first one. This comes from the very particular form of exponential utilities. □

A.4. Optimality verification

In this paragraph, we use the notations U_F and U_B to have more concise expressions. So, it is necessary to check that the solution of this program is an optimum, or equivalently to prove that

$$\mathbb{E}\left[U_F(J(\Theta) - \Theta - \pi\beta_0) - U_F(J^*(\Theta) - \Theta - \pi^*\beta_0)\right] \leq 0$$

or, by introducing the constraint with the Lagrange multiplier coefficient in order to use the first-order conditions

$$\mathbb{E}\begin{bmatrix} U_F(J(\Theta) - \Theta - \pi\beta_0) - U_F(J^*(\Theta) - \Theta - \pi^*\beta_0) \\ +\lambda[U_B(\pi\beta_0 - J(\Theta)) - U_B(\pi^*\beta_0 - J^*(\Theta))] \\ -\lambda[U_B(\pi\beta_0 - J(\Theta)) - U_B(\pi^*\beta_0 - J^*(\Theta))] \end{bmatrix} \leq 0$$

However, as agent B's constraint is satisfied

$$\mathbb{E}\left[U_B(\pi\beta_0 - J(\Theta)) - U_B(\pi^*\beta_0 - J^*(\Theta))\right] \geq 0$$

and

$$-\lambda\mathbb{E}\left[U_B(\pi\beta_0 - J(\Theta)) - U_B(\pi^*\beta_0 - J^*(\Theta))\right] \leq 0$$

Moreover, using the concavity of the utility functions

$$\begin{aligned} &\mathbb{E}\begin{bmatrix} U_F(J(\Theta) - \Theta - \pi\beta_0) - U_F(J^*(\Theta) - \Theta - \pi^*\beta_0) \\ +\lambda[U_B(\pi\beta_0 - J(\Theta)) - U_B(\pi^*\beta_0 - J^*(\Theta))] \\ -\lambda[U_B(\pi\beta_0 - J(\Theta)) - U_B(\pi^*\beta_0 - J^*(\Theta))] \end{bmatrix} \\ &\quad \leq \mathbb{E}\left[(J(\Theta) - J^*(\Theta) + \pi^*\beta_0 - \pi\beta_0)\left\{\begin{matrix} U'_F(J^*(\Theta) - \Theta - \pi^*\beta_0) \\ -\lambda U'_B(\pi^*\beta_0 - J^*(\Theta)) \end{matrix}\right\}\right] \end{aligned} \tag{18}$$

Three cases have to be studied separately

1. If $0 < J^*(\Theta) < \Theta$ $\mathbb{P}$ $a.s.$, condition (7) gives

$$U'_F(J^*(\Theta) - \Theta - \pi^*\beta_0) - \lambda U'_B(\pi^*\beta_0 - J^*(\Theta)) = 0 \quad \mathbb{P}\ a.s.$$

and

$$\mathbb{E}\left[(J(\Theta) - J^*(\Theta) + \pi^*\beta_0 - \pi\beta_0)\left\{\begin{array}{l} U_F'(J^*(\Theta) - \Theta - \pi^*\beta_0) \\ \quad - \lambda U_B'(\pi^*\beta_0 - J^*(\Theta)) \end{array}\right\}\right] = 0$$

Hence the result (using Eq. (18)).

2. If $J^*(\Theta) = 0\ \mathbb{P}\ a.s.$, condition (8) gives

$$U_F'(J^*(\Theta) - \Theta - \pi^*\beta_0) - \lambda U_B'(\pi^*\beta_0 - J^*(\Theta)) \leq 0 \quad \mathbb{P}\, a.s.$$

and

$$\mathbb{E}\left[(J(\Theta) - J^*(\Theta) + \pi^*\beta_0 - \pi\beta_0)\left\{\begin{array}{l} U_F'(J^*(\Theta) - \Theta - \pi^*\beta_0) \\ \quad - \lambda U_B'(\pi^*\beta_0 - J^*(\Theta)) \end{array}\right\}\right] \leq 0$$

Hence the result (using Eq. (18)).

3. If $J^*(\Theta) = \Theta\ \mathbb{P}\ \ a.s.$, condition (9) gives

$$U_F'(J^*(\Theta) - \Theta - \pi^*\beta_0) - \lambda U_B'(\pi^*\beta_0 - J^*(\Theta)) \geq 0 \quad \mathbb{P}\ a.s.$$

and

$$\mathbb{E}\left[(J(\Theta) - J^*(\Theta) + \pi^*\beta_0 - \pi\beta_0)\left\{\begin{array}{l} U_F'(J^*(\Theta) - \Theta - \pi^*\beta_0) \\ \quad - \lambda U_B'(\pi^*\beta_0 - J^*(\Theta)) \end{array}\right\}\right] \leq 0$$

Hence the result (using Eq. (18)).

Hence, the solution is an optimum for the program $(\hat{\mathcal{P}})$.

Acknowledgments

The authors would like to thank Tomas Björk, Monique Jeanblanc, Mathias Tchamengo and two anonymous referees for helpful comments and suggestions, and especially Henri Loubergé for his valuable insights and discussions.

Notes

1. This remark is due to the constructive comments of one anonymous referee on this particular fact.
2. Note that it may also be considered also as the sequence of flows related to an hypothetical non-risky bond with coupon s and principal N and whose price is given by Φ.
3. If the probability measure υ is absolutely continuous with respect to $\mathbb{P}$,

$$h(\upsilon/\mathbb{P}) = \mathbb{E}\left[\frac{d\upsilon}{d\mathbb{P}} \ln\left(\frac{d\upsilon}{d\mathbb{P}}\right)\right]$$

is the relative entropy of υ with respect to $\mathbb{P}$, otherwise

$$h(\upsilon/\mathbb{P}) = +\infty.$$

4. Note that this assumption may be weakened and replaced by an integrability criterion on $\exp(\gamma_I \frac{\alpha^*}{M}\Theta)$.

References

BACHELIER, L. [1900]: "Théorie de la Spéculation," Ph.D. Thesis.

BRIYS, E. [1998a]: "Taming Mother Nature: A Primer on the Pricing of Nature-Linked Securities," Working Paper.

BRIYS, E. [1998b]: "Pricing Mother Nature," *Risk Magazine*, E.P.R.M. (October), 16–18.

CAO, M. and WEI, J. [2000]: "Pricing the Weather," *Risk Magazine* (May), 67–70.

DISCHEL, R. [1998]: "Black-Scholes Won't Do," *Risk Magazine*, E.P.R.M. (October), 8–12.

DISCHEL, R. [1999]: "Shaping History," *Risk Magazine*, E.P.R.M. 13–15.

DISCHEL, R. [2000a]: "Seasonal Weather Forecasts and Derivative Valuation," *Risk Magazine*, E.P.R.M., submitted.

DISCHEL, R. [2000b]: "Is Precipitation Basis Risk Overstated?" *Risk Magazine*, E.P.R.M. (August), 26–27.

DOHERY, N. and MAHUL, O. [2001]: "Mickey Mouse and Moral Hazard: Uninformative but Correlated Triggers," Risk Theory Society, Montreal, Canada (April), 20–22.

DORNIER, F. and QUERUEL, M. [2000]: "Caution to the Wind," *Risk Magazine*, E.P.R.M. (August), 30–32.

EECKHOUDT, L. and GOLLIER, C. [1995]: *Risk: Evaluation, Management and Sharing*. London: Harvester Wheatsheaf.

EL KAROUI, N. and ROUGE, R. [2000]: "Pricing via Utility Maximization and Entropy," *Mathematical Finance*, 10(2), 259–276.

FARNY, D. [1999]: "The Development of European Private Sector Insurance Over the Last 25 Years," *The Geneva Papers on Risk and Insurance*, 24(2) (April), 145–162.

GALLIX, L. [1985]: Il était une fois... l'Assurance: L'Argus Edition.

GEMAN, H. (Ed.) [1999]: *Insurance and Weather Derivatives: From Exotic Options to Exotic Underlyings*. Risk Book.

HODGES, S.D. and NEUBERGER, A. [1989]: "Optimal Replication of Contingent Claims Under Transaction Costs," *Review of Futures Markets*, 8, 222–239.

LONGSTAFF, F. and SCHWARTZ, E. [1995]: "A Simple Approach to Valuing Risky Fixed and Floating Rate Debt," *Journal of Finance*, 50(3) (July), 789–819.

RAVIV, A. [1979]: "The Design of an Optimal Insurance Policy," *American Economic Review*, 69(1) (March), 84–96.

SANKARAN, K. [2000]: "Finding a Value," *Risk Magazine*, E.P.R.M. (August), 21–25.

SOUMELIS FIVOS, D. [1996]: "Assurance Optimale et Asymmetrie de Perception du Risque: Application aux Catastrophes en Milieux Urbains," DEA Dissertation, University of Paris I.

TCHAMENGO, M. [1998]: "Stratégies Statiques en Finance," Ph.D. Thesis, University of Bourgogne.